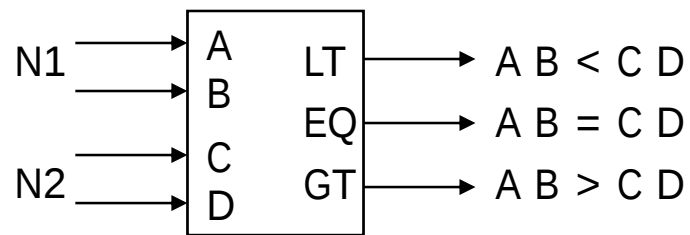


Working with combinational logic

- Simplification
 - two-level simplification
 - exploiting don't cares
 - algorithm for simplification
- Logic realization
 - two-level logic and canonical forms realized with NANDs and NORs
 - multi-level logic, converting between ANDs and ORs
- Time behavior
- Hardware description languages

Design example: two-bit comparator



block diagram
and
truth table

A	B	C	D	LT	EQ	GT
0	0	0	0	0	1	0
		0	1	1	0	0
		1	0	1	0	0
		1	1	1	0	0
0	1	0	0	0	0	1
		0	1	0	1	0
		1	0	1	0	0
		1	1	1	0	0
1	0	0	0	0	0	1
		0	1	0	0	1
		1	0	0	1	0
		1	1	1	0	0
1	1	0	0	0	0	1
		0	1	0	0	1
		1	0	0	0	1
		1	1	0	1	0

we'll need a 4-variable Karnaugh map
for each of the 3 output functions

Design example: two-bit comparator (cont'd)

			A
	0	0	0
	1	0	0
C	1	1	0
	1	1	0
		B	

D

K-map for LT

			A
	1	0	0
	0	1	0
C	0	0	1
	0	0	0
		B	

D

K-map for EQ

			A
	0	1	1
	0	0	1
C	0	0	0
	0	0	1
		B	

D

K-map for GT

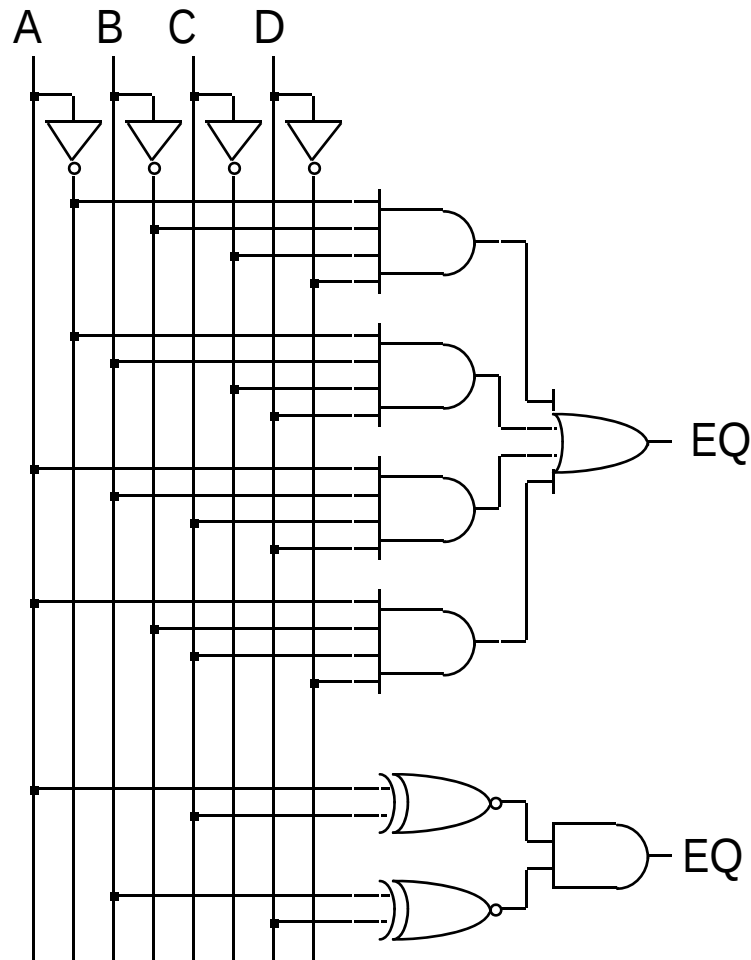
$$LT = A' B' D + A' C + B' C D$$

$$EQ = A' B' C' D' + A' B C' D + A B C D + A B' C D' = (A \text{ xnor } C) \cdot (B \text{ xnor } D)$$

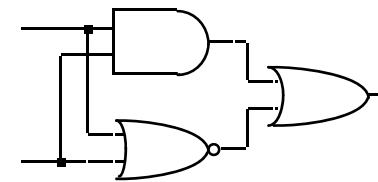
$$GT = B C' D' + A C' + A B D'$$

LT and GT are similar (flip A/C and B/D)

Design example: two-bit comparator (cont'd)

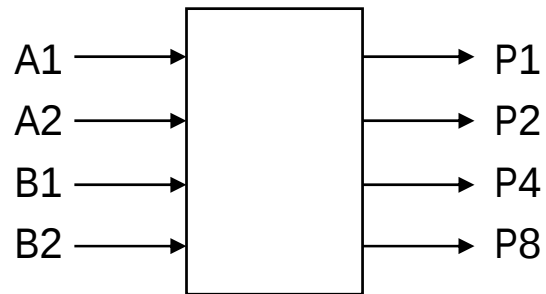


two alternative
implementations of EQ
with and without XOR



XNOR is implemented with
at least 3 simple gates

Design example: 2x2-bit multiplier

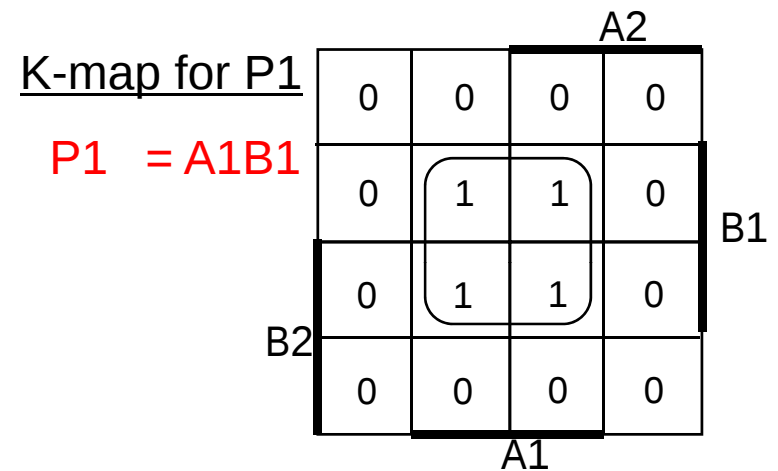
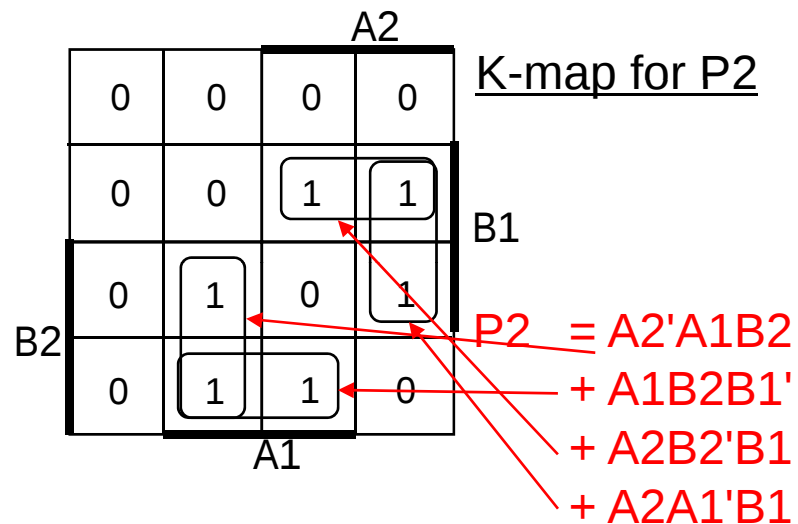
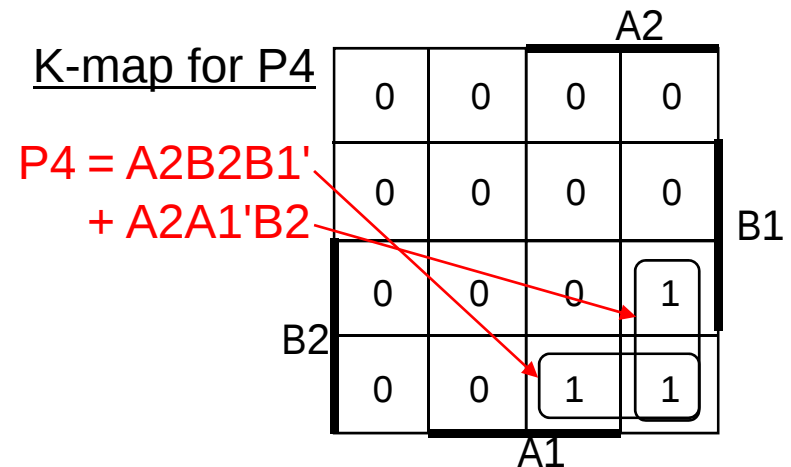
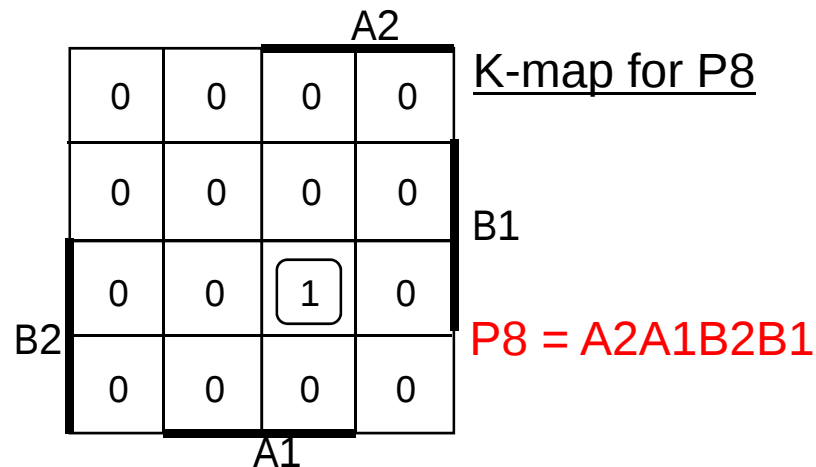


block diagram
and
truth table

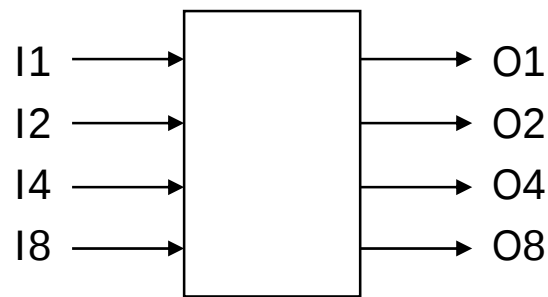
A2	A1	B2	B1	P8	P4	P2	P1
0	0	0	0	0	0	0	0
		0	1	0	0	0	0
		1	0	0	0	0	0
		1	1	0	0	0	0
0	1	0	0	0	0	0	0
		0	1	0	0	0	1
		1	0	0	0	1	0
		1	1	0	0	1	1
1	0	0	0	0	0	0	0
		0	1	0	0	1	0
		1	0	0	1	0	0
		1	1	0	1	1	0
1	1	0	0	0	0	0	0
		0	1	0	0	1	1
		1	0	0	1	1	0
		1	1	1	0	0	1

4-variable K-map
for each of the 4
output functions

Design example: 2x2-bit multiplier (cont'd)



Design example: BCD increment by 1



block diagram
and
truth table

I8	I4	I2	I1	O8	O4	O2	O1
0	0	0	0	0	0	0	1
0	0	0	1	0	0	1	0
0	0	1	0	0	0	1	1
0	0	1	1	0	1	0	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	1	0
0	1	1	0	0	1	1	1
0	1	1	1	1	0	0	0
1	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0
1	0	1	0	X	X	X	X
1	0	1	1	X	X	X	X
1	1	0	0	X	X	X	X
1	1	0	1	X	X	X	X
1	1	1	0	X	X	X	X
1	1	1	1	X	X	X	X

4-variable K-map for each of
the 4 output functions

Design example: BCD increment by 1 (cont'd)

			I8	
	0	0	X	1
	0	0	X	0
I2	0	1	X	X
	0	0	X	X
			I4	
				O8
				I1

$$O8 = I4 I2 I1 + I8 I1'$$

$$O4 = I4 I2' + I4 I1' + I4' I2 I1$$

$$O2 = I8' I2' I1 + I2 I1'$$

$$O1 = I1'$$

			I8	
O4	0	1	X	0
	0	1	X	0
I2	1	0	X	X
	0	1	X	X
			I4	
				I1

			I8	
	0	0	X	0
	1	1	X	0
I2	0	0	X	X
	1	1	X	X
			I4	
				O2
				I1

			I8	
O1	1	1	X	1
	0	0	X	0
I2	0	0	X	X
	1	1	X	X
			I4	
				I1

Definition of terms for two-level simplification

- Implicant
 - ❑ single element of ON-set or DC-set or any group of these elements that can be combined to form a subcube
- Prime implicant
 - ❑ implicant that can't be combined with another to form a larger subcube
- Essential prime implicant
 - ❑ prime implicant is essential if it alone covers an element of ON-set
 - ❑ will participate in ALL possible covers of the ON-set
 - ❑ DC-set used to form prime implicants but not to make implicant essential
- Objective:
 - ❑ grow implicant into prime implicants (minimize literals per term)
 - ❑ cover the ON-set with as few prime implicants as possible (minimize number of product terms)

Examples to illustrate terms

			A	
	0	X	1	0
	1	1	1	0
C	1	0	1	1
	0	0	1	1
			B	

6 prime implicants:

$A'B'D$, BC' , AC , $A'C'D$, AB , $B'CD$

essential

minimum cover: $AC + BC' + A'B'D$

5 prime implicants:

BD , ABC' , ACD , $A'BC$, $A'C'D$

essential

minimum cover: 4 essential implicants

			A	
	0	0	1	0
	1	1	1	0
	0	1	1	1
C	0	1	0	0
			B	

Algorithm for two-level simplification

- Algorithm: minimum sum-of-products expression from a Karnaugh map
 - Step 1: choose an element of the ON-set
 - Step 2: find "maximal" groupings of 1s and Xs adjacent to that element
 - consider top/bottom row, left/right column, and corner adjacencies
 - this forms prime implicants (number of elements always a power of 2)
 - Repeat Steps 1 and 2 to find all prime implicants
 - Step 3: revisit the 1s in the K-map
 - if covered by single prime implicant, it is essential, and participates in final cover
 - 1s covered by essential prime implicant do not need to be revisited
 - Step 4: if there remain 1s not covered by essential prime implicants
 - select the smallest number of prime implicants that cover the remaining 1s

Algorithm for two-level simplification (example)

		A		
	X	1	0	1
	0	1	1	1
C	0	X	X	0
	0	1	0	1
		B		

		A		
	X	1	0	1
	0	1	1	1
C	0	X	X	0
	0	1	0	1
		B		

2 primes around $A'BC'D'$

		A		
	X	1	0	1
	0	1	1	1
C	0	X	X	0
	0	1	0	1
		B		

2 primes around $ABC'D$

		A		
	X	1	0	1
	0	1	1	1
C	0	X	X	0
	0	1	0	1
		B		

3 primes around $AB'C'D'$

		A		
	X	1	0	1
	0	1	1	1
C	0	X	X	0
	0	1	0	1
		B		

2 essential primes

		A		
	X	1	0	1
	0	1	1	1
C	0	X	X	0
	0	1	0	1
		B		

minimum cover (3 primes)

Activity

- List all prime implicants for the following K-map:

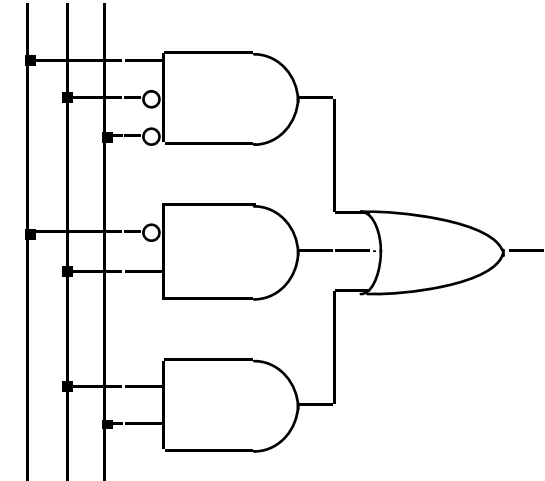
A				D
X	0	X	0	
0	1	X	1	
0	X	X	0	
C	X	1	1	B

- Which are essential prime implicants? --- --- ---
- What is the minimum cover?

Implementations of two-level logic

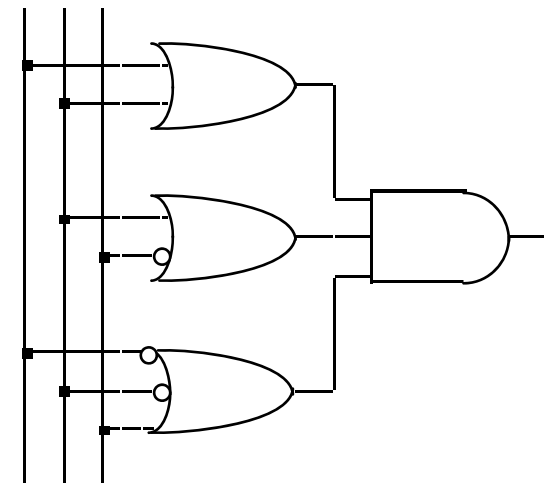
- Sum-of-products

- AND gates to form product terms (minterms)
- OR gate to form sum



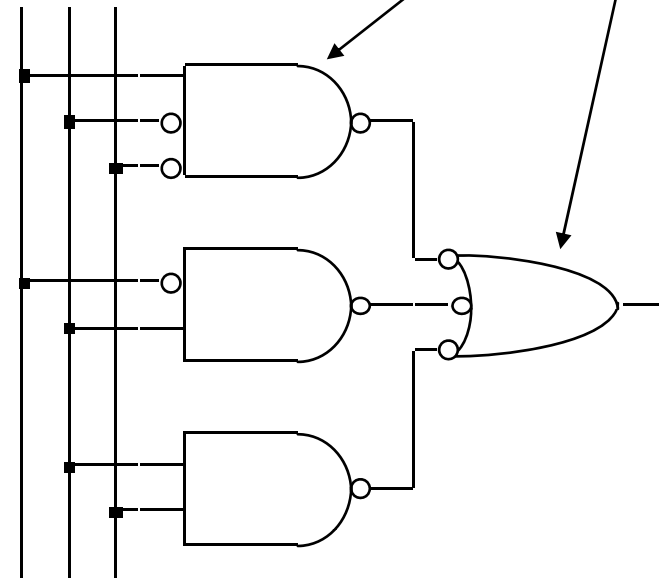
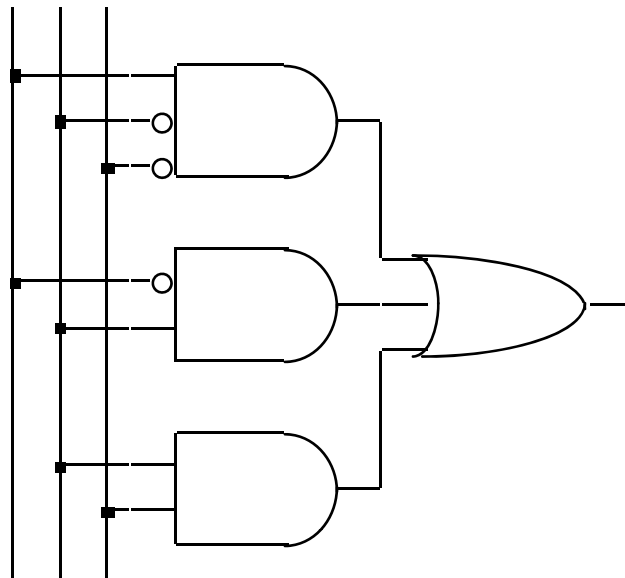
- Product-of-sums

- OR gates to form sum terms (maxterms)
- AND gates to form product



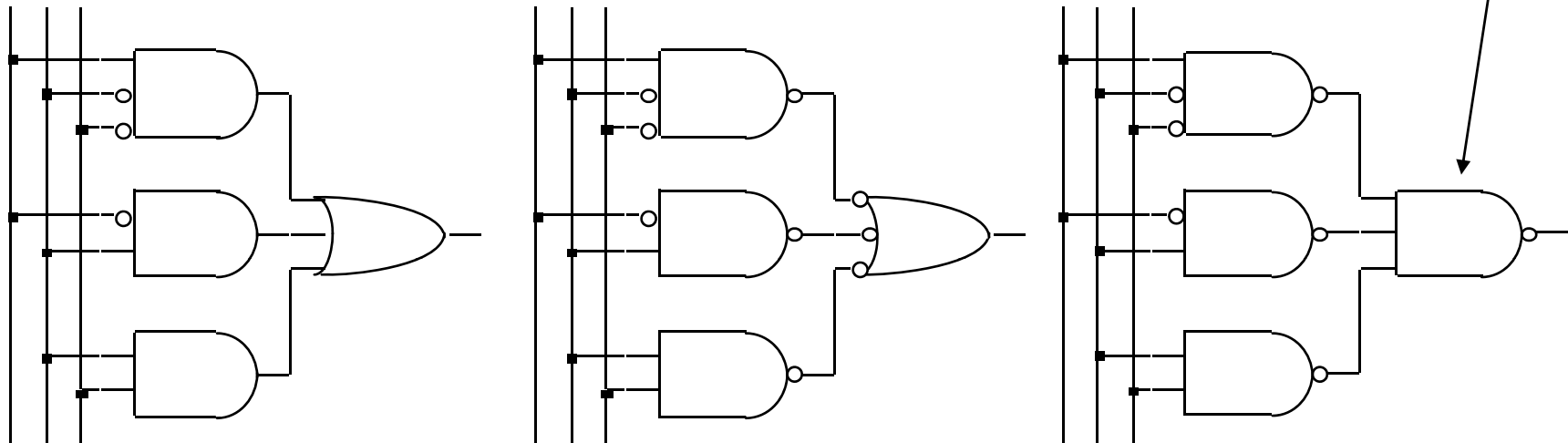
Two-level logic using NAND gates

- Replace minterm AND gates with NAND gates
- Place compensating inversion at inputs of OR gate



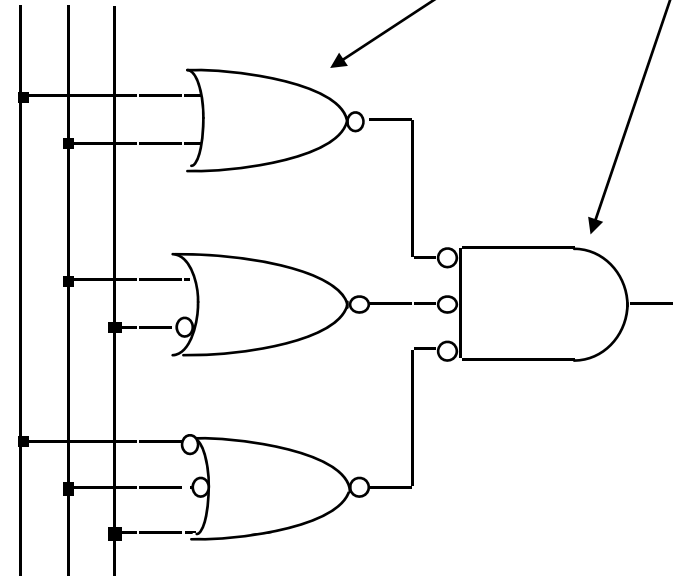
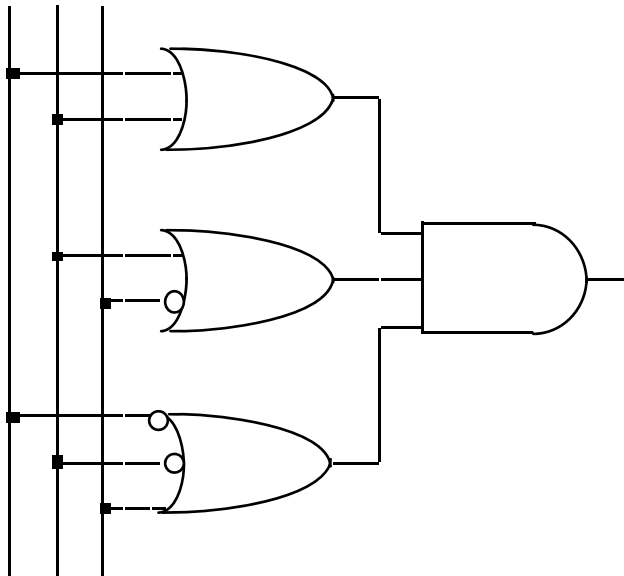
Two-level logic using NAND gates (cont'd)

- OR gate with inverted inputs is a NAND gate
 - de Morgan's: $A' + B' = (A \cdot B)'$
- Two-level NAND-NAND network
 - inverted inputs are not counted
 - in a typical circuit, inversion is done once and signal distributed



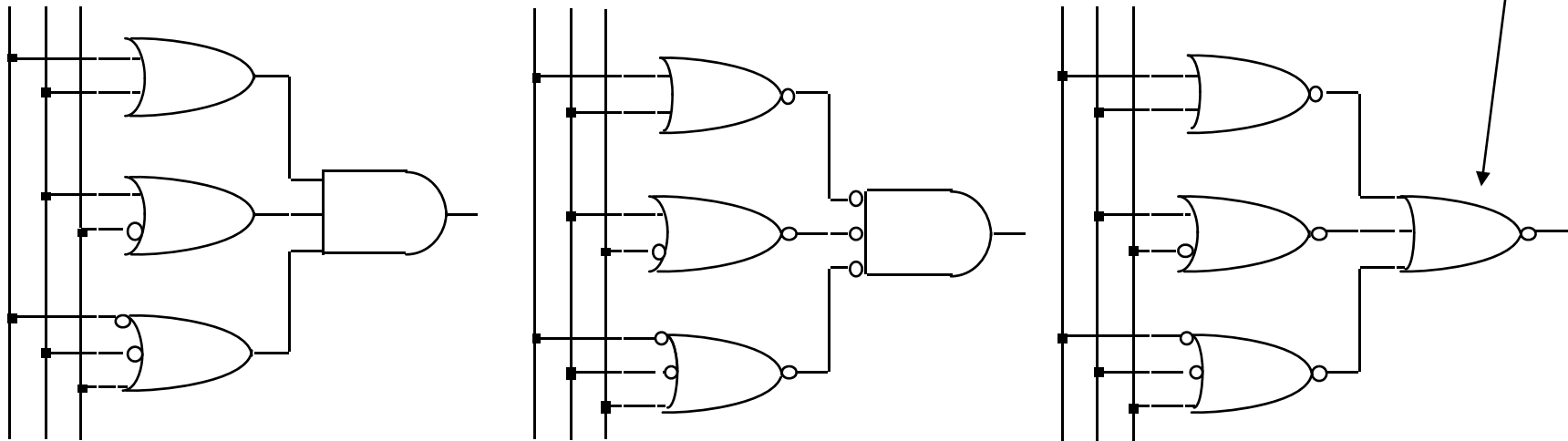
Two-level logic using NOR gates

- Replace maxterm OR gates with NOR gates
- Place compensating inversion at inputs of AND gate



Two-level logic using NOR gates (cont'd)

- AND gate with inverted inputs is a NOR gate
 - de Morgan's: $A' \cdot B' = (A + B)'$
- Two-level NOR-NOR network
 - inverted inputs are not counted
 - in a typical circuit, inversion is done once and signal distributed



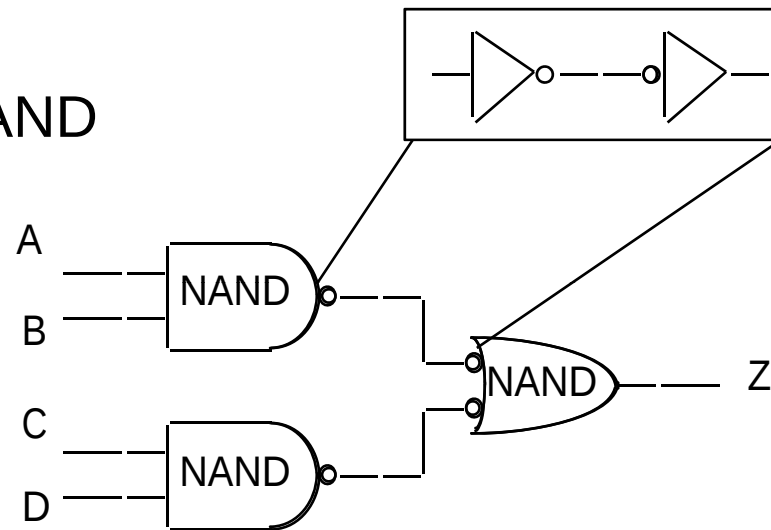
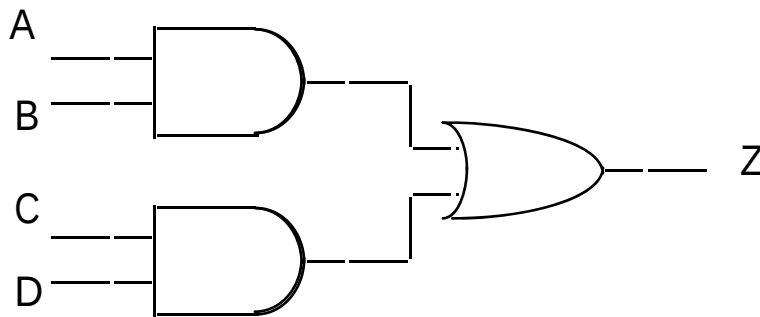
Two-level logic using NAND and NOR gates

- NAND-NAND and NOR-NOR networks
 - de Morgan's law: $(A + B)' = A' \cdot B'$ $(A \cdot B)' = A' + B'$
 - written differently: $A + B = (A' \cdot B')'$ $(A \cdot B) = (A' + B')'$
- In other words —
 - OR is the same as NAND with complemented inputs
 - AND is the same as NOR with complemented inputs
 - NAND is the same as OR with complemented inputs
 - NOR is the same as AND with complemented inputs



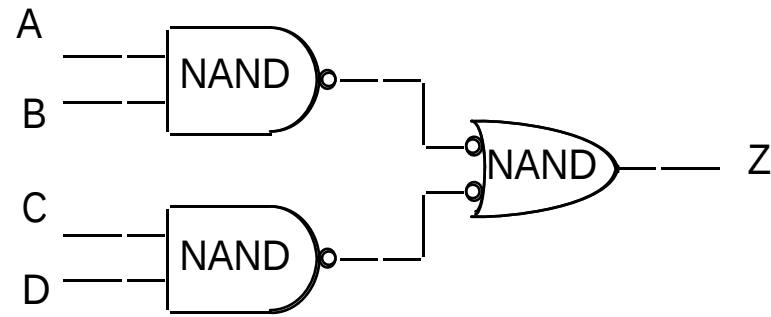
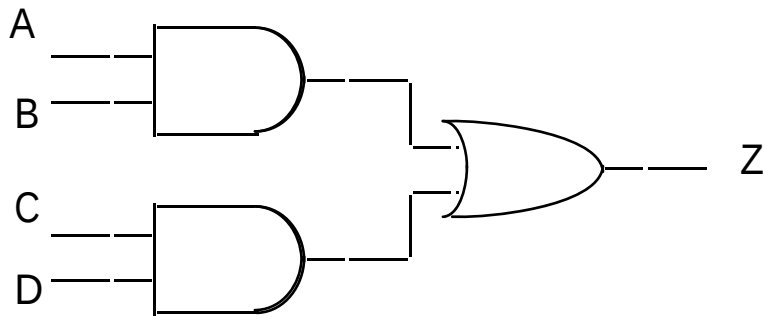
Conversion between forms

- Convert from networks of ANDs and ORs to networks of NANDs and NORs
 - introduce appropriate inversions ("bubbles")
- Each introduced "bubble" must be matched by a corresponding "bubble"
 - conservation of inversions
 - do not alter logic function
- Example: AND/OR to NAND/NAND



Conversion between forms (cont'd)

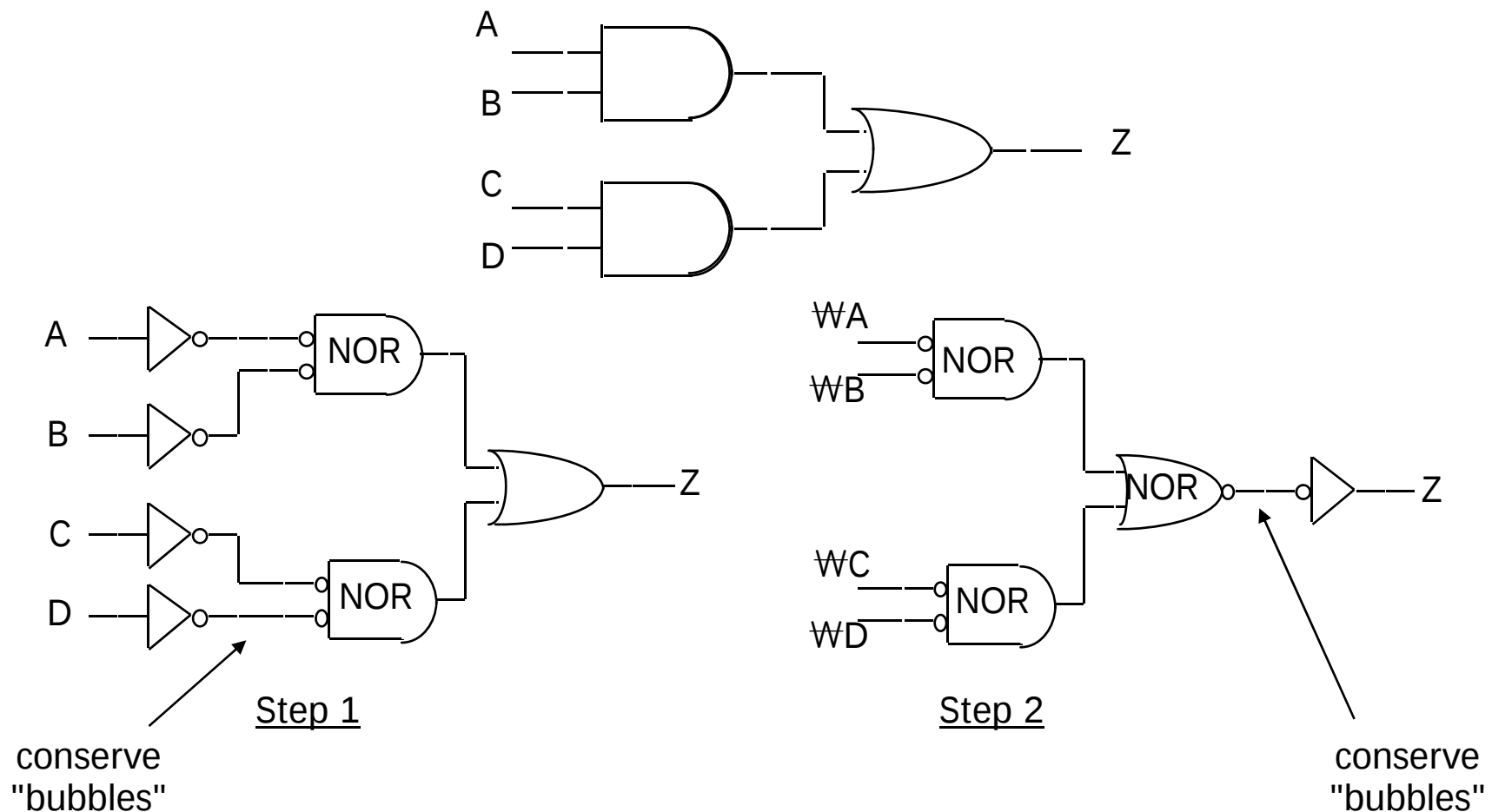
- Example: verify equivalence of two forms



$$\begin{aligned} Z &= [(A \cdot B)' \cdot (C \cdot D)']' \\ &= [(A' + B') \cdot (C' + D')]' \\ &= [(A' + B')' + (C' + D')'] \\ &= (A \cdot B) + (C \cdot D) \quad \checkmark \end{aligned}$$

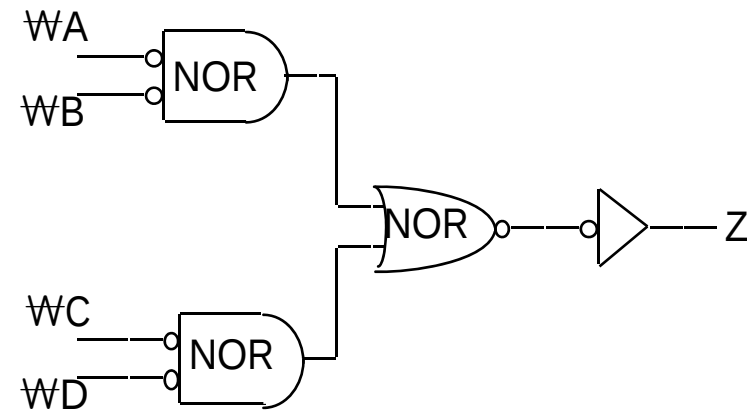
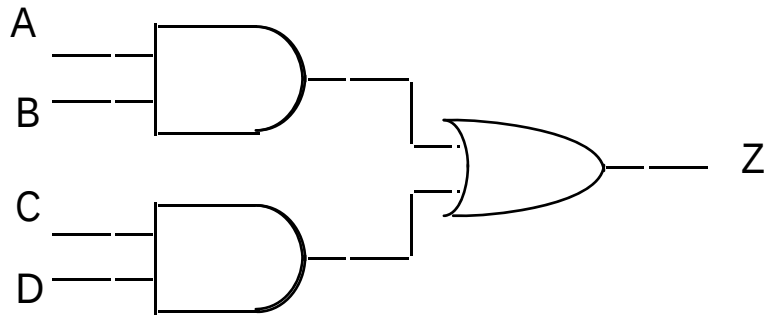
Conversion between forms (cont'd)

- Example: map AND/OR network to NOR/NOR network



Conversion between forms (cont'd)

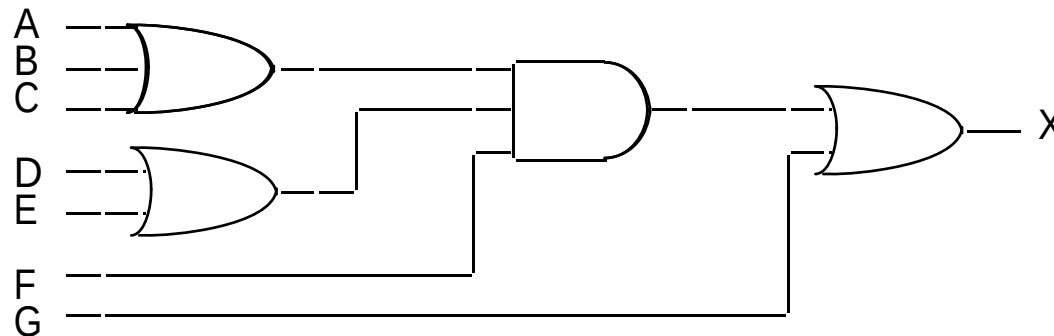
- Example: verify equivalence of two forms



$$\begin{aligned} Z &= \{ [(A' + B')' + (C' + D')']' \}' \\ &= \{ (A' + B') \cdot (C' + D') \}' \\ &= (A' + B')' + (C' + D')' \\ &= (A \cdot B) + (C \cdot D) \quad \checkmark \end{aligned}$$

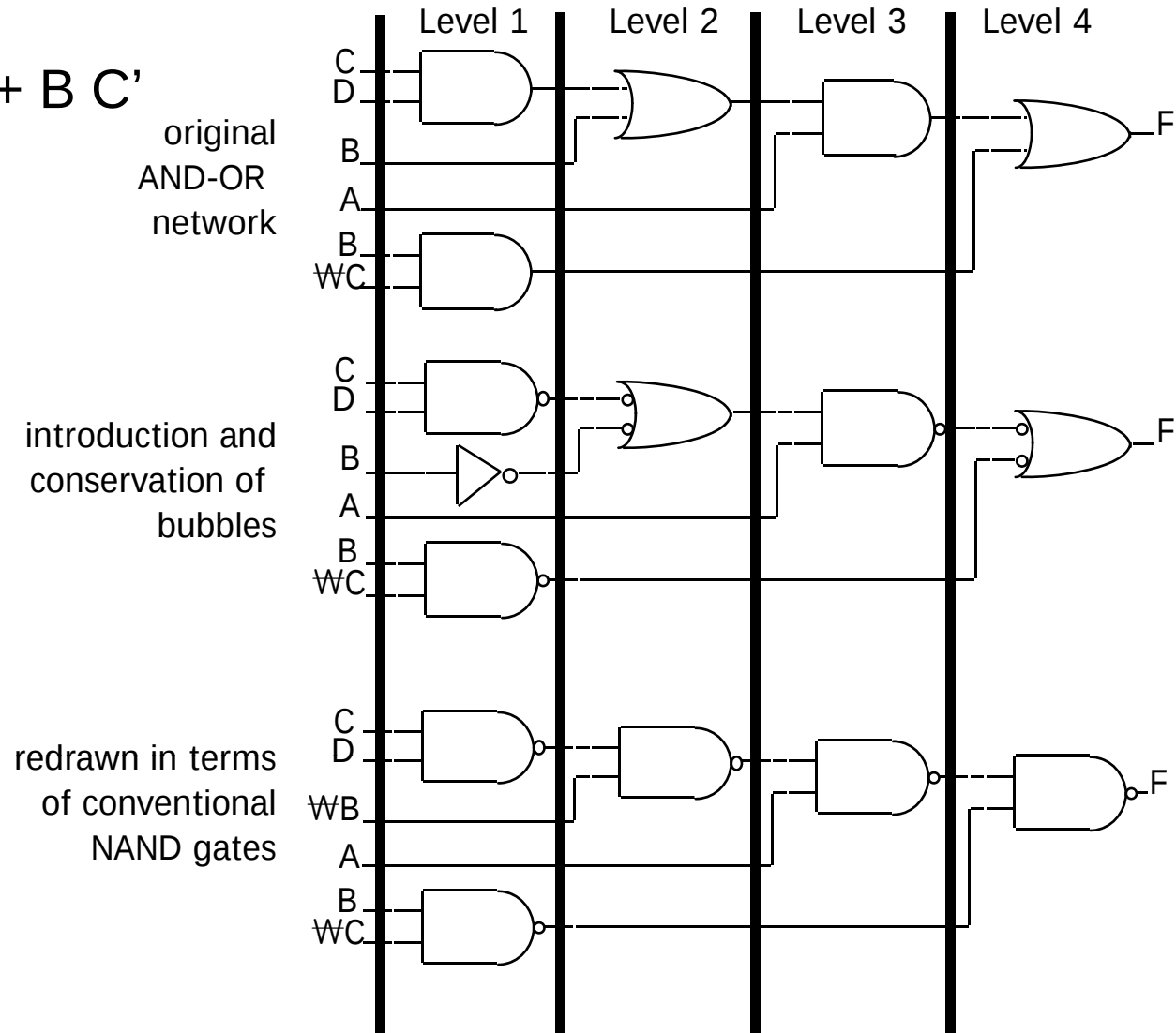
Multi-level logic

- $x = A D F + A E F + B D F + B E F + C D F + C E F + G$
 - ❑ reduced sum-of-products form – already simplified
 - ❑ 6 x 3-input AND gates + 1 x 7-input OR gate (that may not even exist!)
 - ❑ 25 wires (19 literals plus 6 internal wires)
- $x = (A + B + C) (D + E) F + G$
 - ❑ factored form – not written as two-level S-o-P
 - ❑ 1 x 3-input OR gate, 2 x 2-input OR gates, 1 x 3-input AND gate
 - ❑ 10 wires (7 literals plus 3 internal wires)



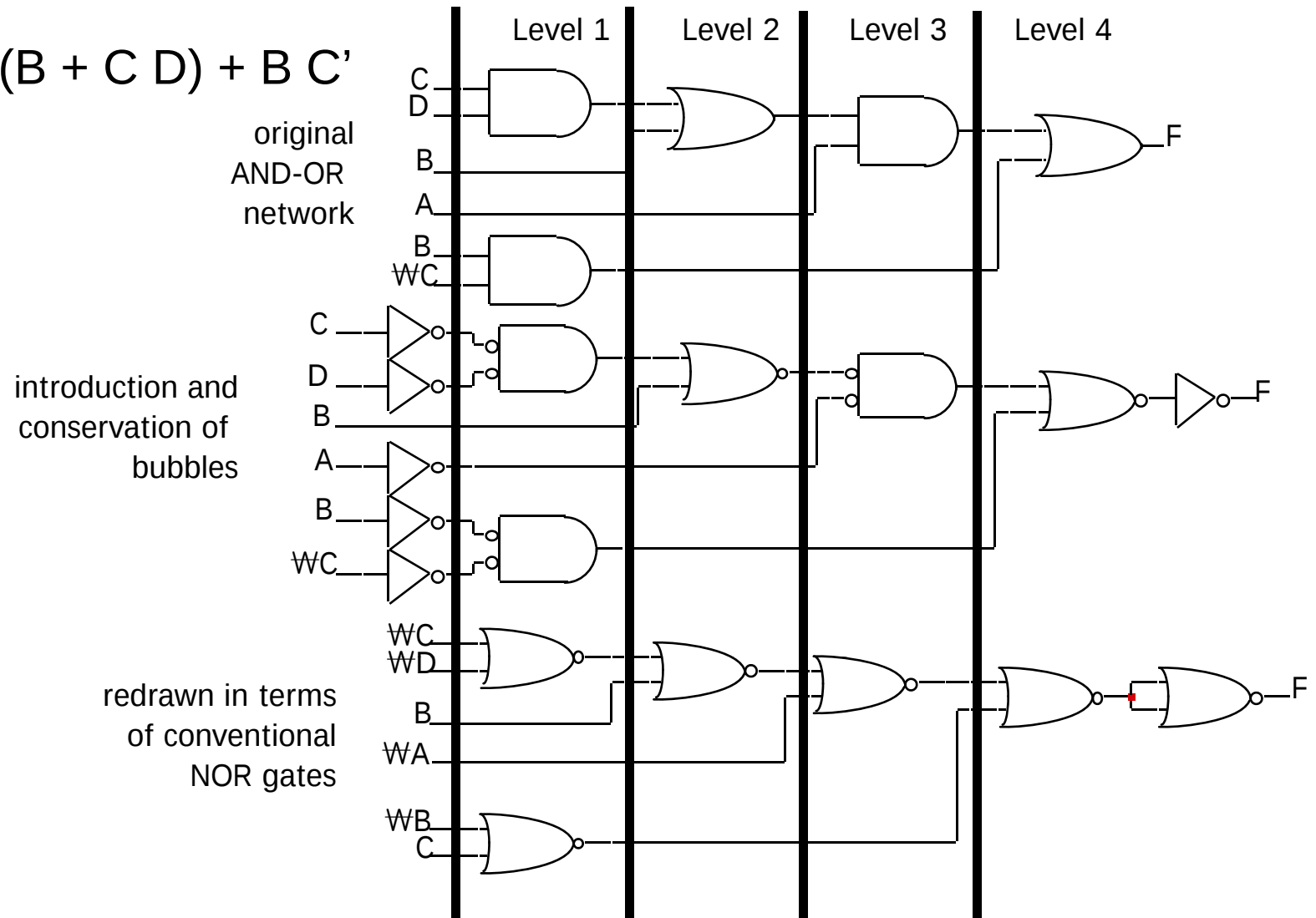
Conversion of multi-level logic to NAND gates

■ $F = A(B + CD) + B C'$



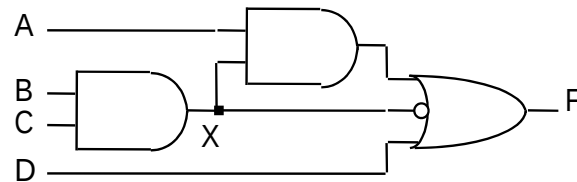
Conversion of multi-level logic to NORs

■ $F = A(B + CD) + BC'$

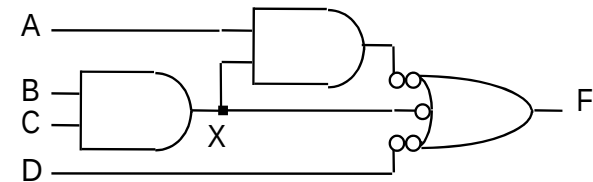


Conversion between forms

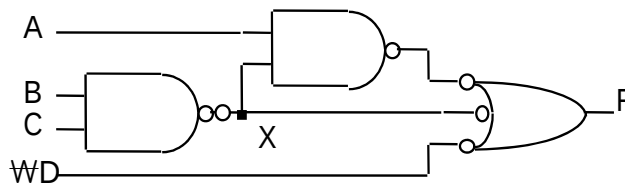
■ Example



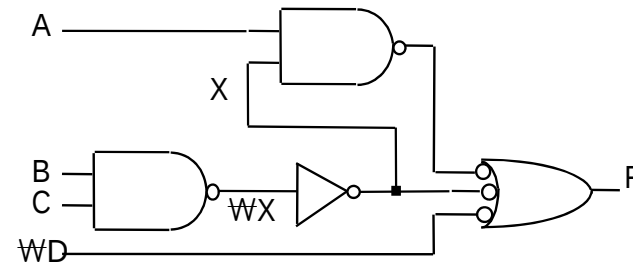
original circuit



add double bubbles to
invert all inputs of OR gate



add double bubbles to
invert output of AND gate

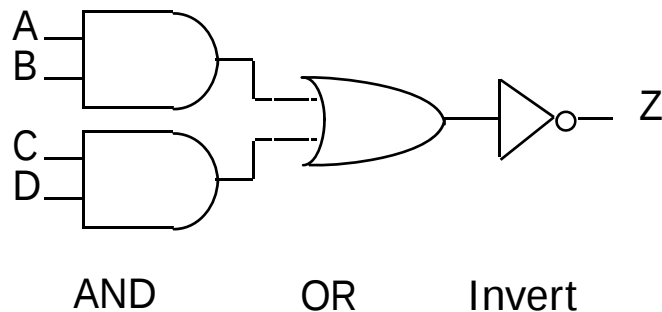


insert inverters to eliminate
double bubbles on a wire

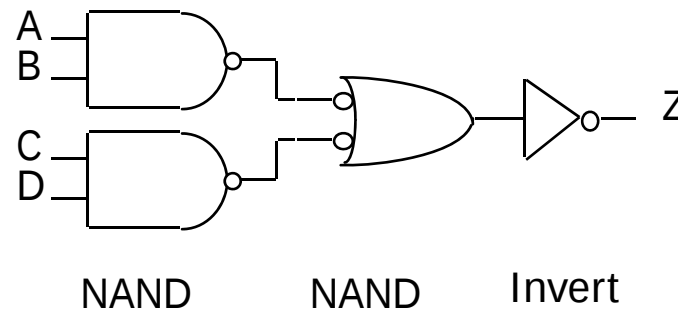
AND-OR-invert gates

- AOI function: three stages of logic — AND, OR, Invert
 - multiple gates "packaged" as a single circuit block

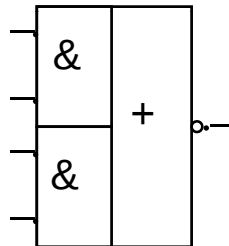
logical concept



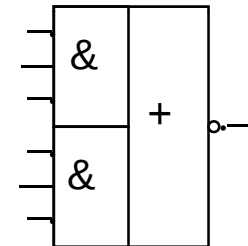
possible implementation



2x2 AOI gate symbol

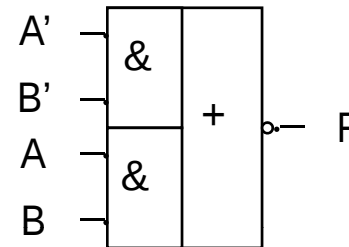


3x2 AOI gate symbol



Conversion to AOI forms

- General procedure to place in AOI form
 - compute the complement of the function in sum-of-products form
 - by grouping the 0s in the Karnaugh map
- Example: XOR implementation
 - $A \text{ xor } B = A' B + A B'$
 - AOI form:
 - $F = (A' B' + A B)'$



Examples of using AOI gates

■ Example:

- $F = A B + A C' + B C'$
- $F = (A' B' + A' C + B' C)'$
- Implemented by 2-input 3-stack AOI gate

- $F = (A + B) (A + C') (B + C')$
- $F = [(A' + B') (A' + C) (B' + C)]'$
- Implemented by 2-input 3-stack OAI gate

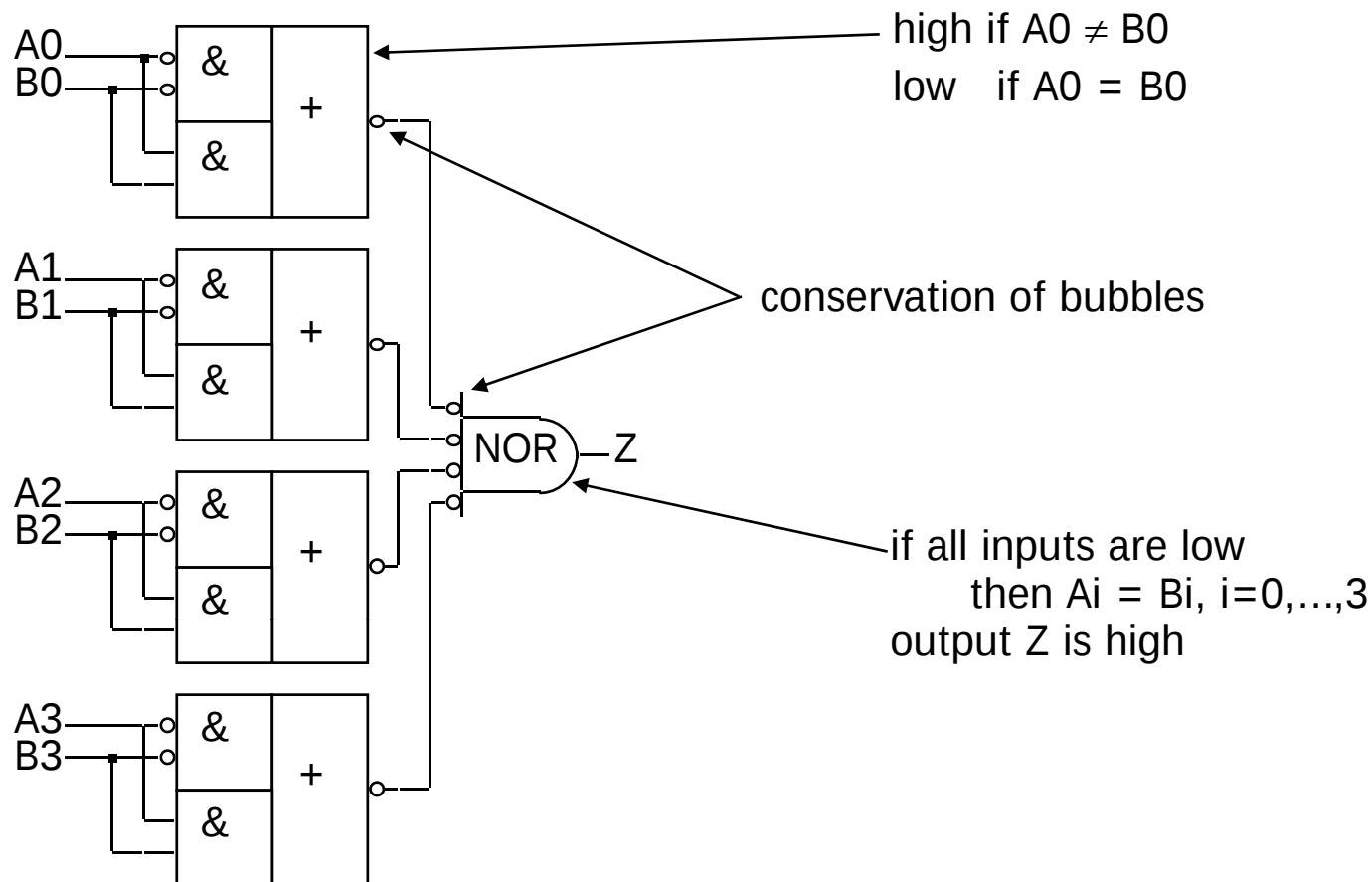
■ Example: 4-bit equality function

- $Z = (A_0 B_0 + A_0' B_0')(A_1 B_1 + A_1' B_1')(A_2 B_2 + A_2' B_2')(A_3 B_3 + A_3' B_3')$

each implemented in a single 2x2 AOI gate

Examples of using AOI gates (cont'd)

- Example: AOI implementation of 4-bit equality function



Summary for multi-level logic

■ Advantages

- ❑ circuits may be smaller
- ❑ gates have smaller fan-in
- ❑ circuits may be faster

■ Disadvantages

- ❑ more difficult to design
- ❑ tools for optimization are not as good as for two-level
- ❑ analysis is more complex

Time behavior of combinational networks

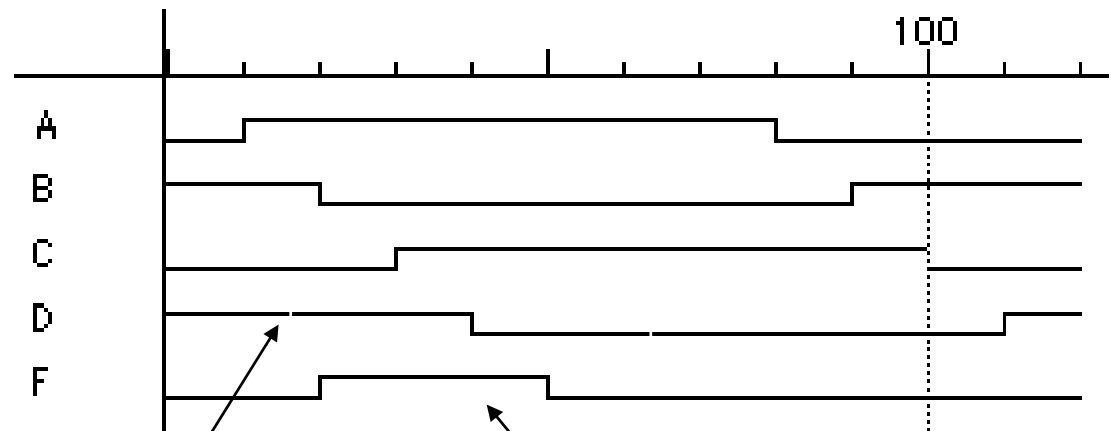
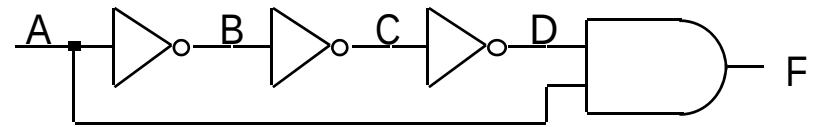
- Waveforms
 - visualization of values carried on signal wires over time
 - useful in explaining sequences of events (changes in value)
- Simulation tools are used to create these waveforms
 - input to the simulator includes gates and their connections
 - input stimulus, that is, input signal waveforms
- Some terms
 - gate delay — time for change at input to cause change at output
 - min delay – typical/nominal delay – max delay
 - careful designers design for the worst case
 - rise time — time for output to transition from low to high voltage
 - fall time — time for output to transition from high to low voltage
 - pulse width — time that an output stays high or stays low between changes

Momentary changes in outputs

- Can be useful — pulse shaping circuits
- Can be a problem — incorrect circuit operation (glitches/hazards)
- Example: pulse shaping circuit

□ $A' \cdot A = 0$

□ delays matter

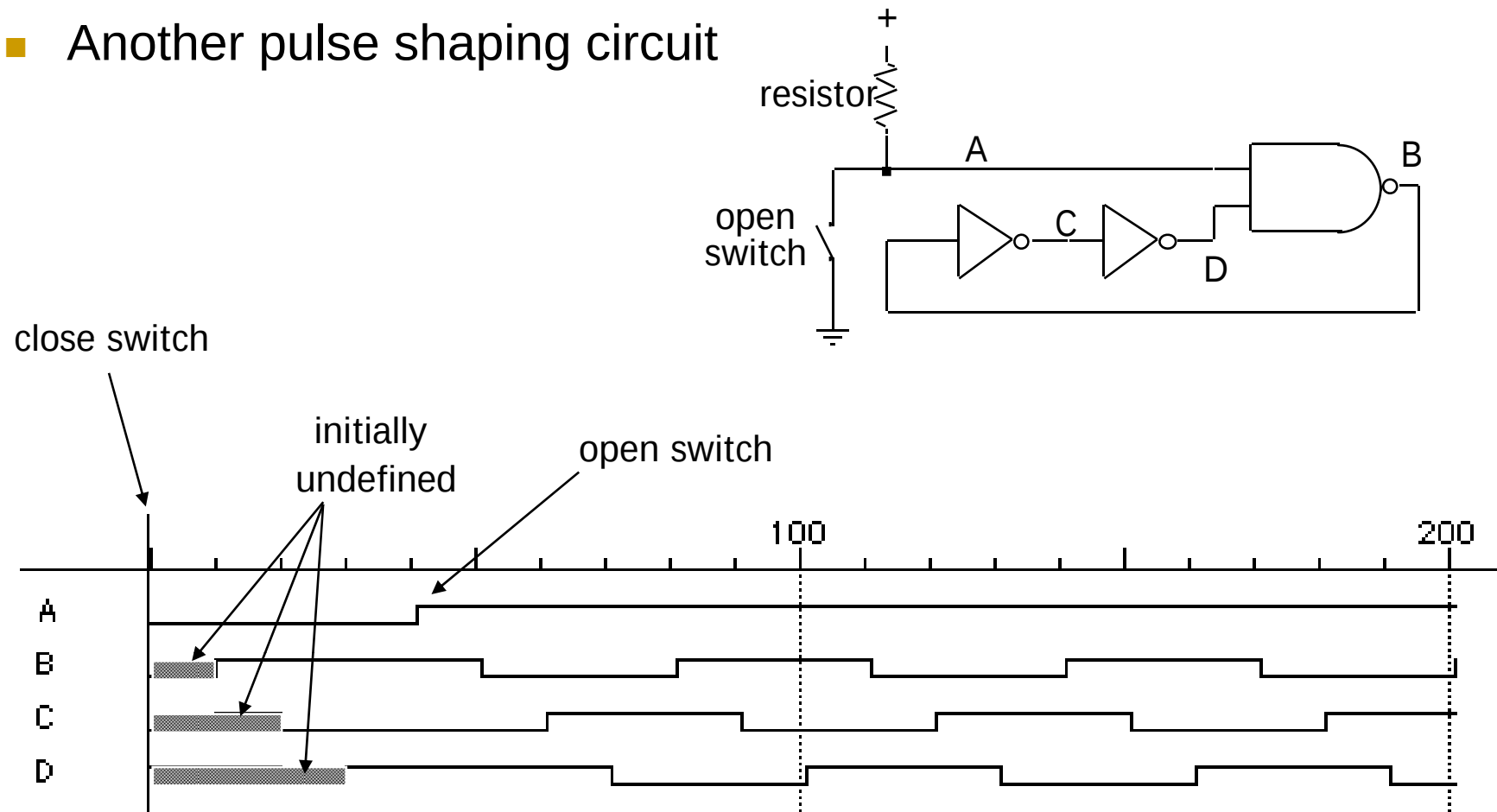


D remains high for
three gate delays after
A changes from low to high

F is not always 0
pulse 3 gate-delays wide

Oscillatory behavior

■ Another pulse shaping circuit



Hardware description languages

- Describe hardware at varying levels of abstraction
- Structural description
 - textual replacement for schematic
 - hierarchical composition of modules from primitives
- Behavioral/functional description
 - describe what module does, not how
 - synthesis generates circuit for module
- Simulation semantics

HDLs

- Abel (circa 1983) - developed by Data-I/O
 - ❑ targeted to programmable logic devices
 - ❑ not good for much more than state machines
- ISP (circa 1977) - research project at CMU
 - ❑ simulation, but no synthesis
- Verilog (circa 1985) - developed by Gateway (absorbed by Cadence)
 - ❑ similar to Pascal and C
 - ❑ delays is only interaction with simulator
 - ❑ fairly efficient and easy to write
 - ❑ IEEE standard
- VHDL (circa 1987) - DoD sponsored standard
 - ❑ similar to Ada (emphasis on re-use and maintainability)
 - ❑ simulation semantics visible
 - ❑ very general but verbose
 - ❑ IEEE standard

Verilog

- Supports structural and behavioral descriptions
- Structural
 - explicit structure of the circuit
 - e.g., each logic gate instantiated and connected to others
- Behavioral
 - program describes input/output behavior of circuit
 - many structural implementations could have same behavior
 - e.g., different implementation of one Boolean function
- We'll mostly be using behavioral Verilog in Aldec ActiveHDL
 - rely on schematic when we want structural descriptions

Structural model

```
module xor_gate (out, a, b);  
    input      a, b;  
    output     out;  
    wire       abar, bbar, t1, t2;  
  
    inverter invA (abar, a);  
    inverter invB (bbar, b);  
    and_gate and1 (t1, a, bbar);  
    and_gate and2 (t2, b, abar);  
    or_gate  or1 (out, t1, t2);  
  
endmodule
```

Simple behavioral model

- Continuous assignment

```
module xor_gate (out, a, b);  
  input          a, b;  
  output         out;  
  reg            out;  
  
  assign #6 out = a ^ b;  
  
endmodule
```

simulation register -
keeps track of
value of signal

delay from input change
to output change

Simple behavioral model

- always block

```
module xor_gate (out, a, b);  
    input        a, b;  
    output       out;  
    reg          out;  
  
    always @(a or b) begin  
        #6 out = a ^ b;  
    end  
  
endmodule
```

specifies when block is executed
ie. triggered by which signals

Driving a simulation through a “testbench”

```
module testbench (x, y);  
    output          x, y;  
    reg [1:0]      cnt;  
  
    initial begin  
        cnt = 0;  
        repeat (4) begin  
            #10 cnt = cnt + 1;  
            $display ("@ time=%d, x=%b, y=%b, cnt=%b",  
                    $time, x, y, cnt); end  
            #10 $finish;  
        end  
  
        assign x = cnt[1];  
        assign y = cnt[0];  
    endmodule
```

2-bit vector

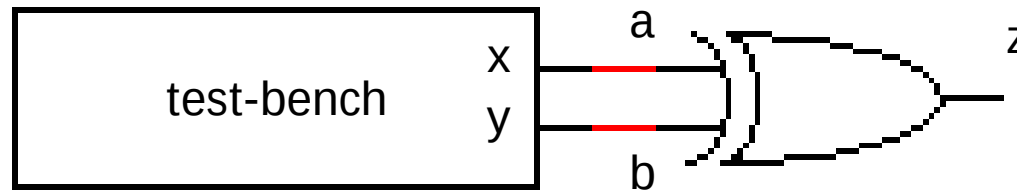
initial block executed only once at start of simulation

print to a console

directive to stop simulation

Complete simulation

- Instantiate stimulus component and device to test in a schematic



Comparator example

```
module Compare1 (Equal, Alarger, Blarger, A, B);  
    input      A, B;  
    output     Equal, Alarger, Blarger;  
  
    assign #5 Equal = (A & B) | (~A & ~B);  
    assign #3 Alarger = (A & ~B);  
    assign #3 Blarger = (~A & B);  
endmodule
```

More complex behavioral model

```
module life (n0, n1, n2, n3, n4, n5, n6, n7, self, out);
    input      n0, n1, n2, n3, n4, n5, n6, n7, self;
    output     out;
    reg        out;
    reg [7:0] neighbors;
    reg [3:0] count;
    reg [3:0] i;

    assign neighbors = {n7, n6, n5, n4, n3, n2, n1, n0};

    always @(neighbors or self) begin
        count = 0;
        for (i = 0; i < 8; i = i+1) count = count + neighbors[i];
        out = (count == 3);
        out = out | ((self == 1) & (count == 2));
    end

endmodule
```

Hardware description languages vs. programming languages

- Program structure
 - ❑ instantiation of multiple components of the same type
 - ❑ specify interconnections between modules via schematic
 - ❑ hierarchy of modules (only leaves can be HDL in Aldec ActiveHDL)
- Assignment
 - ❑ continuous assignment (logic always computes)
 - ❑ propagation delay (computation takes time)
 - ❑ timing of signals is important (when does computation have its effect)
- Data structures
 - ❑ size explicitly spelled out - no dynamic structures
 - ❑ no pointers
- Parallelism
 - ❑ hardware is naturally parallel (must support multiple threads)
 - ❑ assignments can occur in parallel (not just sequentially)

Hardware description languages and combinational logic

- Modules - specification of inputs, outputs, bidirectional, and internal signals
- Continuous assignment - a gate's output is a function of its inputs at all times (doesn't need to wait to be "called")
- Propagation delay- concept of time and delay in input affecting gate output
- Composition - connecting modules together with wires
- Hierarchy - modules encapsulate functional blocks

Working with combinational logic summary

- Design problems
 - filling in truth tables
 - incompletely specified functions
 - simplifying two-level logic
- Realizing two-level logic
 - NAND and NOR networks
 - networks of Boolean functions and their time behavior
- Time behavior
- Hardware description languages
- Later
 - combinational logic technologies
 - more design case studies